MATH 1552 Final Review

Use properties of limits to evaluate the following:

1. $\lim\limits_{x\to-9} \dfrac{x^2-81}{x+9}$

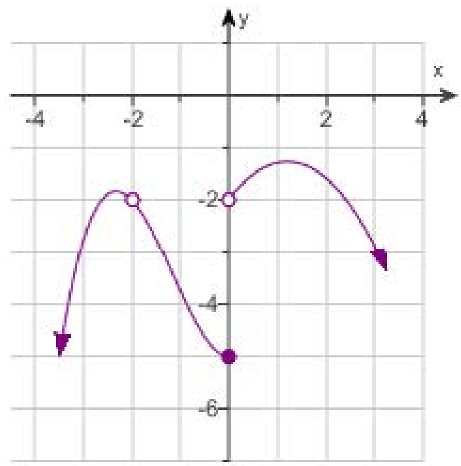
2. $\lim\limits_{x\to 6} \dfrac{x^2-5x-6}{x-6}$

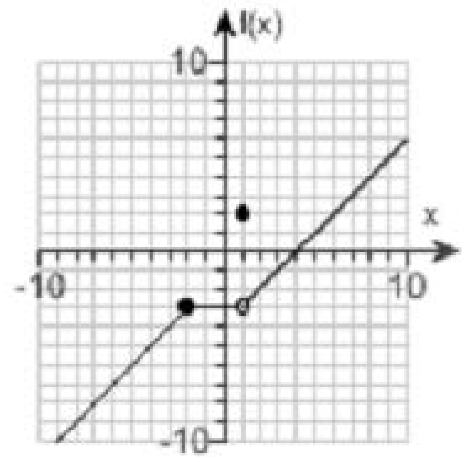
3. $\lim\limits_{x\to 7} \dfrac{\sqrt{x}-\sqrt{7}}{x-7}$

4. $\lim\limits_{x\to\infty} \dfrac{2x^2}{3x^x+19}$

5. $\lim\limits_{x\to\infty} \dfrac{2x^5+9x^3-20}{8x^6-x^4+x^2}$

Use the graphs of the functions to find the following limits:





6. (a) $\lim\limits_{x\to-2} f(x)$
   (b) $\lim\limits_{x\to 0} f(x)$

7. (a) $\lim\limits_{x\to-2} f(x)$
   (b)$\lim\limits_{x\to 1} f(x)$

8. Find the average rate of change of the function $f(x) = x^2 + 3x - 10$ between $x = 2$ and $x = 4$.

9. Find the instantaneous rate of change of the function $f(x) = x^2 - 6x + 1$ when $x = 1$.

Find the derivative of the functions: (10-28)

10. $y = x^3 - 12x^2 + 35x - 500$

11. $y = x^4 - \dfrac{x^3}{45} + x + 29$

12. $f(x) = 5x^{0.4} - 6x^{2.1}$

13. $y = \dfrac{8}{x^3} - \dfrac{3}{x}$

14. $y = \dfrac{1}{x^4} - \dfrac{3}{x^3} + \dfrac{6}{x} + \sqrt{10}$

15. $y = \dfrac{-18}{\sqrt[3]{x}}$

16. $y = (x+4)(3x+1)$

17. $y = \dfrac{x+1}{x+2}$

18. $y = (x^2+1)(2x-3)$

19. $y = \dfrac{3x-1}{x-3}$

20. $y = (2x^3 - x^2 + 8)^5$

21. $y = -4(3x^2+16)^{-3}$

22. $s(t) = 20(9t^3-8)^{\frac{3}{4}}$

23. $f(x) = 12\sqrt{3x^2+10}$

24. $m(x) = 4x(6x^3-1)^5$

25. $y = \dfrac{-9}{(2x^4+1)^3}$

26. $y = -4e^{-2x+9}$

27. $y = -6e^{4x^3}$

28. $y = 3e^{x^2+10}$

29. Find $f'(-2)$ if $f(x) = \dfrac{x^3}{12} - 14x$

30. Find the derivative of y with respect to x if $y = e^{-10x}$

For the following functions, find $f''(x)$ and then find the given values. (31-32)

31. $f(x) = 4x^3 - 9x^2 + 4x + 12$; $f''(0)$ and $f''(3)$

32. $f(x) = \dfrac{x^2}{5+x}$; $f''(0)$ and $f''(9)$

33. For the function, $f(x) = x^5 - 5x^4 + 7x^3 - 12x^2 + 20x - 31$, find $f^{(3)}(x)$ and $f^{(4)}(x)$

34. Find the slope and the equation of the tangent line to the graph of the function at the given value of x. $f(x) = x^3 - 24x^2 - 112x - 114$ at $x = -2$

35. Find all the values of x where the tangent line is horizontal. $f(x) = 2x^3 - 33x^2 + 168x - 104$

36. Find all the points on the graph of $f(x) = 10x^2 - 32x + 25$ where the slope of the tangent line is zero.

37. Find the slope of the tangent line to the graph of the given function at the given value of x. $f(x) = 15x^{\frac{1}{2}} + x^{\frac{3}{2}}$; $x = 9$

For the following functions, find: (38-42) a) the critical numbers b) the open intervals where the function is increasing c) the open intervals where theh function is decreasing

38. $f(x) = 3.1 + 4.1x - 0.6x^2$

39. $f(x) = \dfrac{4}{3}x^3 - 5x^2 - 126x + 40$

40. $f(x) = 12x^3 - 45x^2 - 756x + 30$

41. $f(x) = \sqrt{x^2+9}$

42. $f(x) = (x-3)e^{-2x}$

Find the x-values of all the points where the function has any relative extrema. Find the values of any relative extrema.

43. $f(x) = x^2 - 9x - 10$

44. $f(x) = -\frac{2}{3}x^3 - \frac{11}{2}x^2 - 5x + 11$

45. $f(x) = x^3 - 3x^2 - 45x - 40$

46. $f(x) = x^4 - 162x^2 + 3196$

47. $f(x) = 4 + (7 + 5x)^{\frac{3}{5}}$

48. $f(x) = x - \frac{25}{x}$

49. $f(x) = x^2e^x - 4$

Find the open intervals where the function is concave upward or concave downward. Find any inflection points. (50-52)

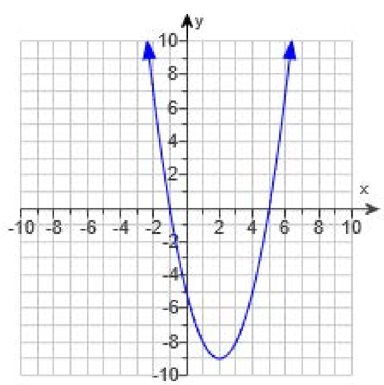
50. $f(x) = -5x^2 - 4x + 13$

51. $f(x) = -\frac{1}{3}x^3 + x^2 + 29x - 6$

52. $f(x) = \frac{4}{x - 7}$

Find the open intervals where the function graphed is a) increasing, b) decreasing. Find the locations and values of all relative extrema. (53-55)

53.



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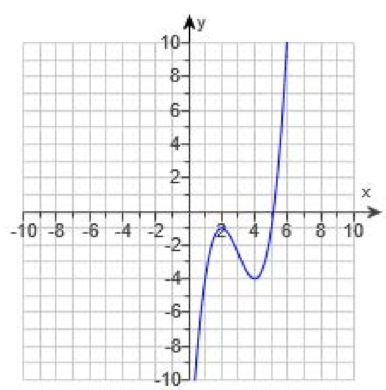

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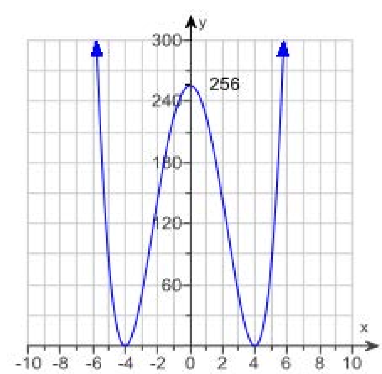


Graph the function, considering the domain, critical points, symmetry, regions where the function is increasing or decreasing, inflection points, regions where the function is concave upward or concave downward, intercepts where possible, and asymptotes where applicable. (56-58)

56. $f(x) = -6x^3 - 18x^2 + 144x - 10$

57. $f(x) = x^4 - 54x^2 + 104$

58. $f(x) = -5x - \frac{35}{x}$

59. Find the absolute maximum and minimum values of $f(x) = 2x^3 + 5x^2 - 4x + 3$ over the interval [0,1]. Also, indicate the x-values which they occur.

60. Find the absolute maximum and minimum values of $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 20x + 9$ over the interval [-2,5]. Also, indicate the x-values which they occur.

61. Find the absolute maximum and minimum values of $f(x) = x^4 - 50x^2 + 20$ on the domain [-6,6].

62. Find the absolute extrema if they exist, as well as all the values of x where they occur, for the function $f(x) = \dfrac{4-x}{7+x}$ on the domain [0,8].

63. Find the absolute extrema if they exist, as well as all the values of x where they occur, for the function $f(x) = \dfrac{-2x}{x^2+5}$ on the domain [-5,0].

64. Assume that a demand equation is given by $q = 4250 - 50p$. Find the marginal revenue for the production level of 3400 units (value of q). (Hint: Solve the demand equation for p and use $R(q) = qp$)

65. If the price in dollars of a stereo system is given by $p = \dfrac{3500}{q^2} + 4200$, where q represents the demand for the product, find the marginal revenue when the demand is 40.

66. The sales of a new item (in thousands) is given by $S(t) = 72e^{-0.3t}$ where t represents time in years. Find the rate of change of sales a) after 1 year, b) after 5 years, c) what is happening to the rate of change as times goes on?

67. A campground owner has 3400 m or fencing. He wants to enclose a rectangular field bordering a river, with no fencing along the river. Let x represent the width of the field.

    (a) Write an expression for the length of the field as a function of x.
    (b) Find the area of the field as a function of x.
    (c) Find the value of x leading to the maximum area.
    (d) Find the maximum area.

68. The total profit $P(x)$ from the sale of x items is approximated by $P(x) = -x^3+6x^2+231x-600$, for $x \geq 4$. Find the number of items that must be sold to maximize profit. Find the maximum profit.

69. Find the point of diminishing returns (x, y) for the function $R(x)$, where $R(x)$ represents the revenue and x represents the amount spent in advertising. $R(x) = 14500 - x^3 + 81x^2 + 700x$, for $0 \leq x \leq 30$

Evaluate the following indefinite integrals: (70-76)

70. $\displaystyle\int 14dx$

71. $\displaystyle\int 10x + 9dx$

72. $\displaystyle\int 12x^5 + 6x^2 - 8x + 13dx$

73. $\displaystyle\int 14\sqrt{x} + 12dx$

74. $\displaystyle\int 6x^4(x^3+10)dx$

75. $\displaystyle\int \frac{1}{5x^5}dx$

76. $\displaystyle\int 9e^{-0.6x}dx$

Use substitution to find the following. (77-80)

77. $\int 5(3x-5)^2 dx$

78. $\int \frac{30}{(6x+1)^5} dx$

79. $\int x^6 e^{4x^7} dx$

80. $\int (x+3)e^{x^2+6} dx$

Evaluate the definite integral. (81-84)

81. $\int_4^9 7dx$

82. $\int_3^7 2x-5dx$

83. $\int_0^1 5x^2-10x+6dx$

84. $\int_0^3 10\sqrt{8x+1}dx$

Find the cost functions for the given marginal cost functions and fixed costs. (85-86)

85. $C'(x) = 9x - 15$; given the fixed cost is 70

86. $C'(x) = 0.007e^{0.02x}$; given the fixed cost is 5

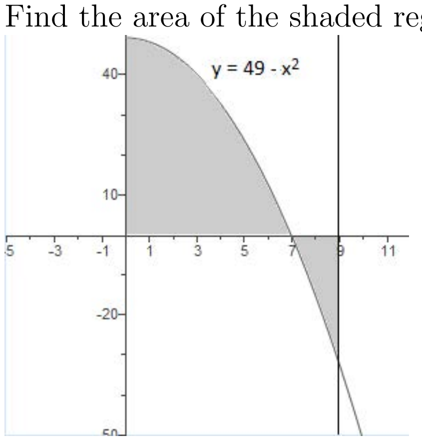
Use the definite integral to find the area between the x-axis and $f(x)$ over the indicated interval. Check first to see if the graph crosses the x-axis in the given interval. (87-89)

87. $f(x) = 4x - 13$; [2,5]

88. $f(x) = 75 - 3x^2$; [3,9]

89. $f(x) = e^x - 3$; [0,4]

90. Find the area of the shaded region.



91. A company found that its rate of profit (in thousands of dollars) after t years of operation is given by the function $P'(t) = (8t+12)(t^2+3t+5)^{\frac{1}{3}}$

(a) find the total profit in the first three years

(b) Find the profit in the fourth year of operation

(c) What is happening to the annual profit over the long run?

Find the area between the given curves. (92-94)

92. $x = -6$, $x = 2$, $y = 2x^2 + 8$, $y = 0$

93. $x = -3$, $x = 3$, $y = x^2 - 7$, $y = 6x$

94. $y = x^2 - 30$, $y = 5 - 2x$

95. Suppose a company wants to introduce a new machine that will produce a rate of annual savings (in dollars) given by the function $S'(x)$, where x is the number of years of operation of the machine, while producing a rate of annual costs (in dollars) given by the function $C'(x)$ $S'(x) = \frac{224}{3} - x^2$, $C'(x) = x^2 - \frac{20}{3}x$

   (a) For how many years will it be profitable to use this new machine?

   (b) What are the nest total savings during the first year of use of this machine?

   (c) What are the net total savings over the period of use of this machine?

96. Find the producers' surplus if the supply function of a commodity is $S(q) = q^3 - 3q + 45.639$. Assume supply and demand are in equilibrium at $q = 2.1$

97. The supply and demand functions (in dollars) are given by $S(q) = q^2 + 18q$ and $D(q) = 793 - 17q - q^2$

   (a) Sketch a graph of the supply and demand curves.

   (b) Find the point at which supply and demand are in equilibrium.

   (c) Find the consumers' surplus

   (d) Find the producers' surplus

98. Find the compound amount for the deposit and the amount of interest earned for \$650 at 6.4% compounded semiannually for 17 years.

99. Find the compound amount for the deposit and the amount of interest earned for \$6300 at 4% compounded quarterly for 9 years.

100. Find the present value (the amount that should be invested now) to accumulate \$4066.23 at 7.2% compounded quarterly for 6 years.

101. Find the interest rate for a \$2000 deposit accumulating to 3889.14, compounded quarterly for 8 years.

102. Find the effective rate corresponding to each nominal rate compounded as indicated.

   (a) 4.1% quarterly

   (b) 6.3% monthly

   (c) 7.4% semiannually

   (d) 5% continuously

103. Find the future value of an ordinary annuity if payments are made in the amount of \$730 at 6% interest compounded quarterly for 15 years.

104. Find the amount of the payment to be made into a sinking fund so that enough will be present to accumulate \$85,000 if the money earns 6% compounded semiannually for 10 years. Payments are made at the end of each period.

105. Find the present value of the ordinary annuity which has payments of \$2500 per year for 9 years at 5% compounded annually.

106. Find the payments necessary to amortize a 9% loan of \$54,000 compounded monthly for 10 years. Also, find the total payment and the amount of interest paid.